Graph Theory

Graph theory helps in solving real-world problems efficiently, making it essential in technology and science

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Introduction

Graph theory is the study of networks of connected objects. A graph consists of **nodes (vertices)** and **edges (connections)**. If edges have a **direction**, it's a **directed graph (digraph)**; otherwise, it's **undirected**.

Why is Graph Theory Important?

1. **Navigation & Routing** - Used in GPS systems, internet routing, and traffic optimization.

2. **Social Networks** - Helps analyze connections and influence, like in Facebook or Twitter.

3. **Data Relationships** - Useful in databases, recommendation systems (Netflix, Amazon), and web linking (Google's PageRank).

4. **Biology & Chemistry** - Helps model DNA structures, chemical compounds, and disease spread.

5. **Artificial Intelligence** - Used in neural networks, decision trees, and search algorithms.

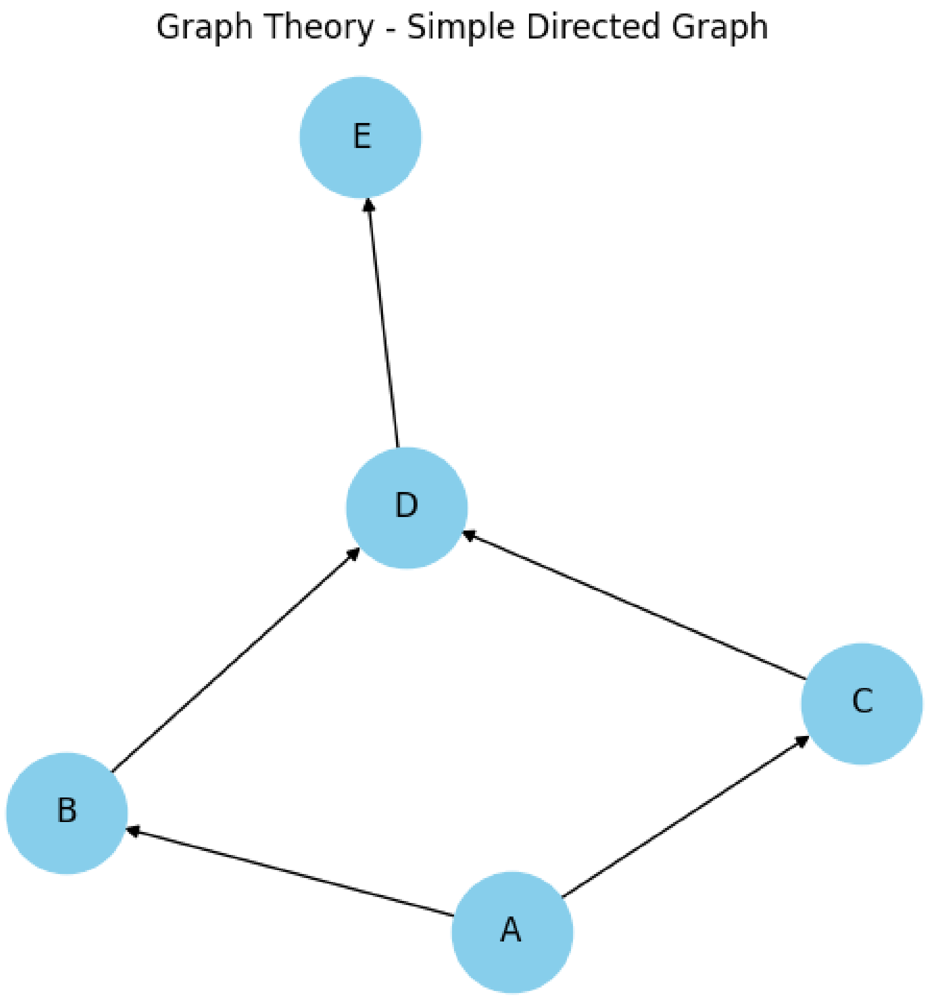
6. **Project Management** - Critical path analysis in workflows and task dependencies.

```python
import networkx as nx
import matplotlib.pyplot as plt

# Create a directed graph
G = nx.DiGraph()

# Add edges (connections)
edges = [
    ("A", "B"),
    ("A", "C"),
    ("B", "D"),
    ("C", "D"),
    ("D", "E")
]
G.add_edges_from(edges)

# Draw the graph
plt.figure(figsize=(5, 5))
nx.draw(
    G,
    with_labels=True,
    node_color="skyblue",
    edge_color="black",
    arrows=True,
    node_size=2000,
    font_size=12)
plt.title("Graph Theory - Simple Directed Graph")
plt.show()
```



Applications

Question 1: Shortest Path in a Road Network (Dijkstra's Algorithm)

A logistics company called Home Logistics wants to determine the most efficient route between two cities in a given road network. The network is represented as a graph where cities are nodes and roads are edges with weights corresponding to the travel distance (in kilometers).

Given the following graph representation of a road network, write a Python program using Dijkstra's Algorithm to find the shortest path from City A to City F.

Graph Data (as adjacency list):

```
roads = {
    'A': {'B': 4, 'C': 2},
    'B': {'A': 4, 'C': 5, 'D': 10},
    'C': {'A': 2, 'B': 5, 'D': 3, 'E': 8},
    'D': {'B': 10, 'C': 3, 'E': 6, 'F': 2},
    'E': {'C': 8, 'D': 6, 'F': 4},
    'F': {'D': 2, 'E': 4}
}
```

Answer

```
# Create graph
G = nx.Graph()
edges = [
    ("A", "B", 4),
    ("A", "C", 2),
    ("B", "C", 5),
    ("B", "D", 10),
    ("C", "D", 3),
    ("C", "E", 8),
    ("D", "E", 6),
    ("D", "F", 2),
    ("E", "F", 4)
]
G.add_weighted_edges_from(edges)

# Compute shortest path from A to F
path = nx.shortest_path(
    G,
    source="A",
    target="F",
    weight="weight")
distance = nx.shortest_path_length(
    G,
    source="A",
    target="F",
    weight="weight")

print("Shortest Path:", path)
print("Total Distance:", distance, "km")
```

```
Shortest Path: ['A', 'C', 'D', 'F']
Total Distance: 7 km
```

Question 2: Influence Analysis in a Social Network (PageRank Algorithm)

A social media platform wants to identify the most influential users based on follower relationships. The network is represented as a directed graph, where each user is a node, and an edge from user A to user B means that A follows B. Given the following directed graph of follower relationships, implement a Python program using the PageRank algorithm to rank users by influence.

Graph Representation:

```python
followers = {
    'Alice': ['Bob', 'Charlie'],
    'Bob': ['Charlie', 'David'],
    'Charlie': ['David'],
    'David': ['Alice'],
    'Eve': ['Alice', 'Charlie']
}
```

Compute the PageRank scores and determine the most influential user.

Answer

```python
# Create directed graph
G = nx.DiGraph()
# Graph representation
followers = {
    'Alice': ['Bob', 'Charlie'],
    'Bob': ['Charlie', 'David'],
    'Charlie': ['David'],
    'David': ['Alice'],
    'Eve': ['Alice', 'Charlie']
}
for user, following in followers.items():
    G.add_edges_from((user, f) for f in following)

# Compute PageRank
pagerank_scores = nx.pagerank(G)
print("User Rankings (Most Influential First):")
```

```python
# sort by PageRank score
sorted(pagerank_scores.items(), key=lambda x: x[1], reverse=True)
```

```
User Rankings (Most Influential First):
```

```
[('David', 0.2926192854405204),
 ('Alice', 0.2914779704710639),
 ('Charlie', 0.23202522727386252),
 ('Bob', 0.15387751681455353),
 ('Eve', 0.030000000000000006)]
```

Question 3: Maximum Flow in a Water Distribution System (Ford-Fulkerson Algorithm)

A city's water supply system consists of reservoirs, pipelines, and distribution points. The system is represented as a directed graph, where nodes represent junctions (reservoirs or city areas) and edges represent water pipelines with capacity limits. Given the following network, where the source is S (reservoir) and the sink is T (city distribution center), use the Ford-Fulkerson algorithm to determine the maximum amount of water that can be transported to the city.

Graph Representation (with capacities):

```python
water_network = {
    'S': {'A': 16, 'B': 13},
    'A': {'B': 10, 'C': 12},
    'B': {'D': 14},
    'C': {'B': 9, 'T': 20},
    'D': {'C': 7, 'T': 4},
    'T': {}
}
```

Write a Python program to compute the maximum flow from S to T.

Answer

```python
# Create directed graph with capacities
G = nx.DiGraph()
edges = [
    ("S", "A", 16),
    ("S", "B", 13),
```

```
    ("A", "B", 10),
    ("A", "C", 12),
    ("B", "D", 14),
    ("C", "B", 9),
    ("C", "T", 20),
    ("D", "C", 7),
    ("D", "T", 4)
]
G.add_weighted_edges_from(
    edges,
    weight="capacity")

# Compute max flow from S to T
flow_value, flow_dict = nx.maximum_flow(G, "S", "T")
print("Maximum Flow:", flow_value, "units")
```

```
Maximum Flow: 23 units
```
