Stochastic Programming and Applications

Plot showing the life expectancy using data from the World Health Organization (WHO) between 2000 and 2021 in Kenya.

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Description

Stochastic programming is an optimization framework that incorporates uncertainty into decision-making models. Unlike deterministic optimization, which assumes perfect/fixed information about all parameters, stochastic programming accounts for randomness in constraints and objectives.

Key Features of Stochastic Programming:

1. **Decision Variables**: Represent choices to be optimized.
2. **Uncertainty (Random Variables)**: Captures variability in parameters.

3. **Objective Function**: Typically involves expected value optimization.
4. **Constraints**: Incorporate probabilistic constraints or chance constraints.

Application

The goal is often to **minimize expected cost** or **maximize expected profit** while considering risk measures.

1. **Newsvendor Problem**

   A vendor must decide how many items to stock without knowing the exact demand. The goal is to minimize expected costs, balancing:

   Overstock costs: Money lost on unsold items **Understock costs:** Lost revenue from unmet demand

   Optimal stocking levels depend on the probability distribution of demand.

2. **Portfolio Optimization**

   In finance, investors allocate funds across assets to maximize expected returns while controlling risk. The problem involves:

   Decision variables: Asset allocation weights **Objective:** Maximize expected returns considering the risk **Constraints:** Budget (weights sum to 1), non-negativity, diversification limits.

   The solution balances the trade-off between return and risk across various market scenarios.

Imports

```
import numpy as np
import matplotlib.pyplot as plt
import cvxpy as cp
```

Demand	Probability

Supply Chain Optimization under Demand Uncertainty

Consider Supply Chain Optimization under Demand Uncertainty. Demand for electronic supply in Nairobi in the past two weeks together with their respective probability is given below:

Demand	Probability
255	0.03
302	0.15
270	0.04
317	0.1
285	0.05
332	0.05
300	0.09
347	0.01
315	0.09
362	0.03
330	0.2
262	0.07
309	0.05
277	0.04

Consider `constraints = [supply >= 120, supply <= 330]`. Compute the optimal supply.

```
# Given demand data
demand = np.array([
    255,
    302,
    270,
    317,
    285,
    332,
    300,
    347,
    315,
    362,
    330,
    262,
```

```python
    309,
    277])
probability = np.array([
    0.03,
    0.15,
    0.04,
    0.1,
    0.05,
    0.05,
    0.09,
    0.01,
    0.09,
    0.03,
    0.2,
    0.07,
    0.05,
    0.04])
```

```python
# Verify that probabilities sum to 1
print(f"Sum of probabilities: {np.sum(probability):.2f}")
```

```
Sum of probabilities: 1.00
```

```python
# Define the supply decision variable
supply = cp.Variable()

# Constraints: supply should be between 120 and 330
constraints = [supply >= 120, supply <= 330]
```

```python
# Define the expected deviation cost function (penalizing shortages and excesses)
cost = cp.sum(cp.multiply(probability, cp.abs(supply - demand)))

# Define the optimization problem
problem = cp.Problem(cp.Minimize(cost), constraints)

# Solve the problem
_ = problem.solve()
```

Explanation:

- `cp.abs(supply - demand)`

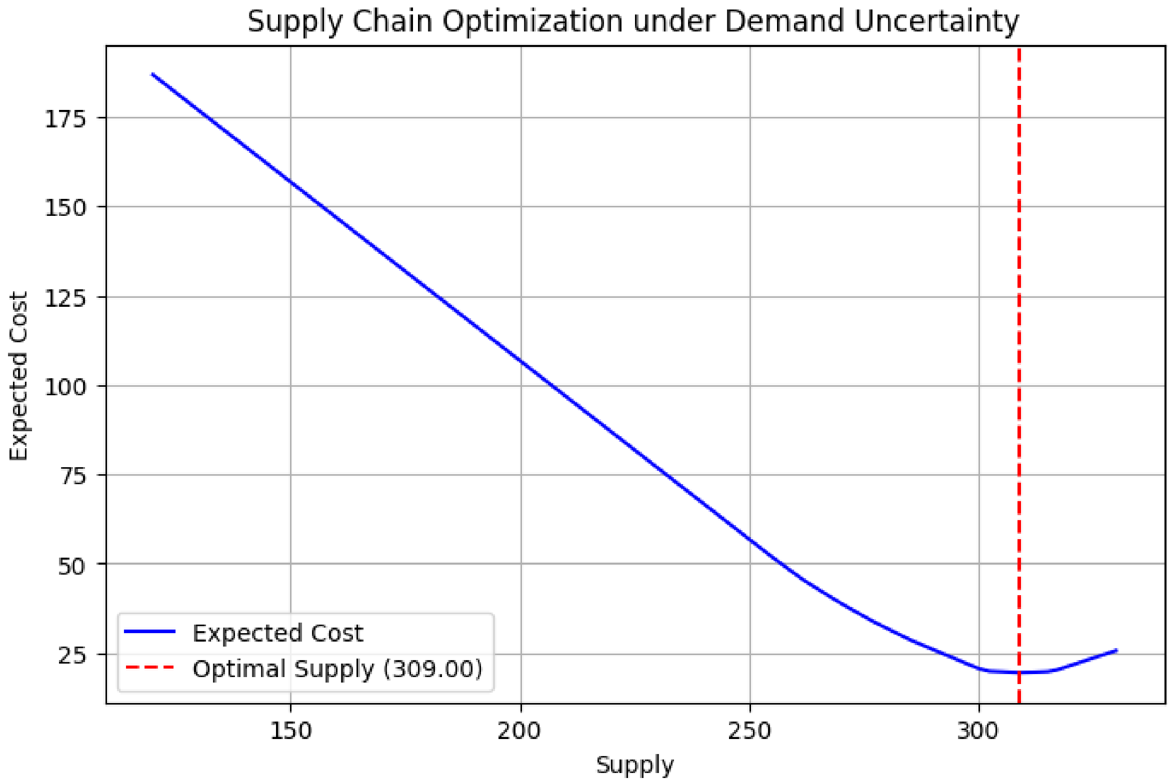
    - This penalizes both over-supply (**excess**) and under-supply (**shortages**), treating both as equally costly.
    - This is useful when deviations in either direction are undesirable.
    - This is our scenario!

- **`cp.pos(supply - demand)`**
    - This only penalizes cases where `supply > demand`, meaning it considers only **excess supply** as costly.
    - It does **not** penalize shortages, which may not be ideal if stockouts are a concern.

```
# Print the optimal supply value
optimal_supply = supply.value
print("Optimal Supply:", optimal_supply)
```

```
Optimal Supply: 309.00000011886954
```

```
# Plot the cost function over a range of supply values
supply_range = np.linspace(120, 330, 100)
cost_values = [sum(p * abs(s - d) for p, d in zip(probability, demand)) for s in supply_range

plt.figure(figsize=(8, 5))
plt.plot(supply_range, cost_values, label='Expected Cost', color='blue')
plt.axvline(optimal_supply, color='red', linestyle='--', label=f'Optimal Supply ({optimal_su
plt.xlabel('Supply')
plt.ylabel('Expected Cost')
plt.title('Supply Chain Optimization under Demand Uncertainty')
plt.legend()
plt.grid()
plt.show()
```



Healthcare Resource Allocation with Uncertain Demand

Consider ICU Bed Allocation under Uncertain Patient Arrivals. Number of unscheduled arrivals at Kenyatta National Hospital in the last 10 days has been `[25, 20, 30, 50, 27, 39, 42, 29, 35, 42]` patients with assigned probabilities `[0.1, 0.1, 0.08, 0.15, 0.09, 0.05, 0.1, 0.1, 0.13, 0.1]` respectively. Consider bed constraints = `[beds >= 17, beds <= 55]`. Compute the optimal number of ICU beds allocation.

```
# Given ICU arrival values and their probabilities
arrivals = np.array([25, 20, 30, 50, 27, 39, 42, 29, 35, 42])
probability = np.array([0.1, 0.1, 0.08, 0.15, 0.09, 0.05, 0.1, 0.1, 0.13, 0.1])
```

```
# Verify that probabilities sum to 1
print(f"Sum of probabilities: {np.sum(probability):.2f}")
```

```
Sum of probabilities: 1.00
```

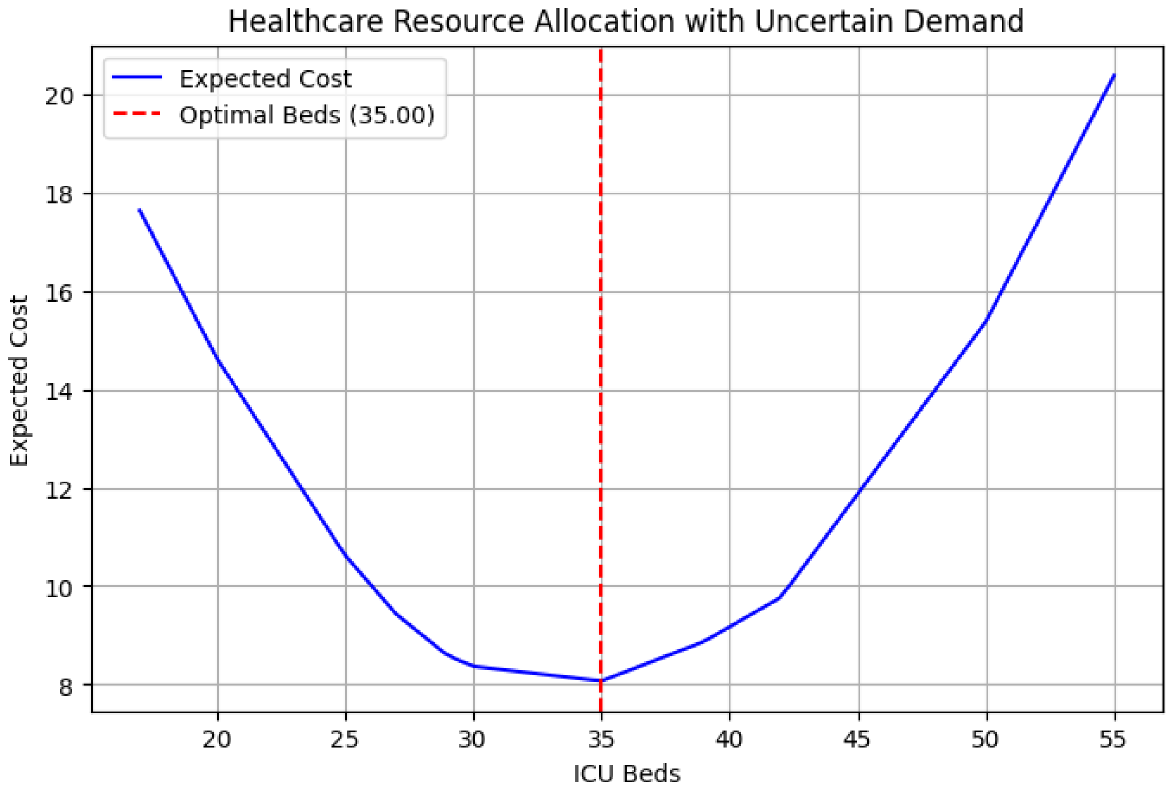
```
# Define the bed allocation decision variable
beds = cp.Variable()

# Constraints: beds should be between 17 and 55
constraints = [beds >= 17, beds <= 55]
```

```
# Define the expected deviation cost function (penalizing shortages and excesses)
cost = cp.sum(cp.multiply(probability, cp.abs(beds - arrivals)))

# Define the optimization problem
problem = cp.Problem(cp.Minimize(cost), constraints)

# Solve the problem
_ = problem.solve()
```

```
# Print the optimal bed allocation
optimal_beds = beds.value
print("Optimal ICU Beds Allocation:", optimal_beds)
```

```
Optimal ICU Beds Allocation: 34.99999972144706
```

```
# Plot the cost function over a range of bed allocations
bed_range = np.linspace(17, 55, 100)
cost_values = [sum(p * abs(b - a) for p, a in zip(probability, arrivals)) for b in bed_range

plt.figure(figsize=(8, 5))
plt.plot(bed_range, cost_values, label='Expected Cost', color='blue')
plt.axvline(optimal_beds, color='red', linestyle='--', label=f'Optimal Beds ({optimal_beds:.
plt.xlabel('ICU Beds')
plt.ylabel('Expected Cost')
plt.title('Healthcare Resource Allocation with Uncertain Demand')
plt.legend()
plt.grid()
plt.show()
```



Healthcare Resource Allocation with Uncertain Demand
Expected Cost
Optimal Beds (35.00)
20
18
16
14
12
10
8
Expected Cost
20
25
30
35
40
45
50
55
ICU Beds